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Multi-group effective cross section calculation method for Fully Ceramic Micro-encapsulated fuel

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ABSTRACT

A multi-group effective cross section calculation method for Fully Ceramic Micro-encapsulated (FCM) fuels containing stochastically dispersed tri-structural isotropic (TRISO) coated fuel particles is proposed to solve the double heterogeneity (DH). In resonance-energy range, the disadvantage factors are obtained by solving a one-dimensional model containing a TRISO particle with hyperfine group method. The matrix and TRISO particles will be homogenized by correcting the hyperfine-group cross sections with disadvantage factors. Due to the large absorption cross section of heavy isotopes in thermal-energy range, the spatial self-shielding effect in the TRISO particles should also be taken account. In the thermal-energy range, the multi-group disadvantage factors are obtained by the neutron's first-collision probabilities and penetrating probability equivalent. Based on the methods described above, the materials in the fuel rod are merged into a homogeneous material. The FCM fuel can be treated as traditional PWR lattice. In the present paper, the Dancoff correction factor of every rod is firstly obtained with neutron current method. Then a one-dimensional model for every fuel rod will be established by Dancoff factor equivalent. Finally, the hyperfine group calculation is carried out based on the one-dimensional rod model to get the effective cross sections of each fuel rod. Numerical results show that the proposed method is proved effective to treat DH for FCM fuel and capable of providing accurate effective cross sections.

1. Introduction

After the Fukushima Daiichi Nuclear Power Plant accident in Japan in March 2011, more and more attentions are attracted to the development of nuclear fuels and claddings with enhance accident tolerance fuel (ATF) in the world. One type of ATFs is the Fully Ceramic Micro-encapsulated fuels adopting tri-structural isotropic particles at PWR conditions (Bragg-Sitton, 2014). The TRISO particles consisting of kernel fissile fuels, protective and shield cell shell materials are stochastically dispersed throughout matrix. The advantage of this kind of fuel is capable of prohibiting fission products from releasing, provided by the silicon carbide (SiC) shell of the TRISO particles. The dense, radiation-resistant SiC matrix where these particles are compacted also serves as the secondary safety boundary.

FCM fuels yield double heterogeneity in the aspect of geometry compared with typical cells in current PWR assembly. The combination of the usual heterogeneity from the fuel element lattice along with the heterogeneous mixture of microspheres constituting the elements yields a doubly heterogeneous environment for particle fuel. Due to the double heterogeneity in FCM fuels, traditional self-shielding calculation method, such as the equivalence theory (Stamm'ler and Abbate, 1983), subgroup method (Nikolaev et al., 1970; Knott and Yamamoto, 2010) and hyperfine group method (Leszczynski, 1987) cannot be directly applied to treat the self-shielding effect in FCM fuels.

Many self-shielding calculation methods have been developed to solve the double heterogeneity in the stochastic media. One method proposed by Hébert (1993) and Sanchez and Pomraning (1991) is based on the transport theory of double heterogeneity system and has been implemented in the DRAGON (Hébert, 2008), HELIOS (Skerjanc et al., 2009) and APOLLO (Sanchez et al., 1988) code. This method establishes neutron slowing-down equation based on dispersed particle fuels and solves it by subgroup method to obtain multi-group effective cross sections. It can exactly describe the randomness of particles, but it is time-consuming, because the 3D neutron slowing-down equation is solved in this method. He et al. (2016) proposed a method where the multi-group effective cross sections in an equivalent one-dimensional spherical shell model is calculated through subgroup method, and then multi-group effective cross sections are

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corrected by Dancoff factors to consider the double heterogeneity effect. There exists another method where the stochastic media will be homogenized by correcting the cross sections with disadvantage factors or self-shielding factors, and the problem can be transformed into traditional geometry with single heterogeneity effect after homogenization. The homogenized geometry with single heterogeneity effect can be solved by conventional self-shielding methods. This method is firstly proposed by the Jülich Research Centre and implemented in the VSOP code (Hansen and Teuchert, 1971). In this code, the self-shielding factors are calculated by penetration probability and collision probabilities equivalence. This method is also adopted in SCALE code (Williams, 2011) where the disadvantage factors are calculated by hyperfine group method based on an equivalent one-dimensional spherical shell model. Recently, Williams et al. (2015) developed a new method to treat resonance self-shielding in doubly heterogeneous very high temperature gas-cooled reactor systems. The method homogenizes the fuel particle and matrix materials using an analytically derived disadvantage factor from a two-region problem with equivalence theory.

In addition to the methods mentioned above, some other methods have also been proposed to treat the double heterogeneity in FCM fuel recently. The resonance self-shielding treatment method based on the embedded self-shielding method (ESSM) is proposed by Li et al. (2018). In this methods, the heterogeneous resonance integral (RI) tables are necessary, which are obtained by the Monte Carlo code. It is very complicated to generate many heterogeneous RI tables to cover all the lattice designs. Another method proposed by Kim et al. (2005) is the reactivity-equivalent physical transformation (RPT) method and it has been applied to FCM fuels (Awan et al., 2017). This method transforms the double-heterogeneous fuel into single-heterogeneous fuel through dividing the FCM fuel lump into two layers. The inside and outside layers are TRISO particles and matrix, respectively. The materials in the two layers are homogenized by volume weight. The radius of inside layers needs to be calculated by preserving the reactivity rate of the realistic problem. The realistic reactivity is obtained by Monte Carlo code. It is inconvenient to provide reference reactivity for various cases using Monte Carlo code.

A precise and efficient multi-group effective cross section calculation method for FCM fuel is proposed in this paper. The double heterogeneity is separated into two parts and solved separately. The first step is to deal with the heterogeneity of TRISO particles. The matrix and TRISO particles will be homogenized by correcting the cross sections with disadvantage factors. The FCM fuel can be treated as current PWR lattice after homogenization. The second step is solving the heterogeneity of the homogenized fuel rods in the lattice system. In the previous work of the authors (Liu et al., 2018), a method is proposed to treat the self-shielding for PWR lattice. In this method, the self-shielding calculation is split into global and local calculations. The global calculations obtain the Dancoff correction factor for each pin cell by neutron current method. Then each pin in the lattice system is isolated from the lattice by establishing an equivalent one-dimensional model which is obtained by preserving Dancoff correction factor. In that work (Liu et al., 2018), the pseudo-resonant-nuclide subgroup method is applied to treat the self-shielding effect in the one-dimensional model. While in the present work, the hyperfine group calculation is adopted to get more accurate effective self-shielded cross sections.

This paper is organized as follows. The methodologies used in the present work are described in Section 2. In Section 3, the proposed method is tested against some DH problems. Some conclusions are given in Section 4.

2. Methodologies

2.1. Overview of the proposed method to treat double heterogeneity effect of FCM fuel

The double heterogeneity of FCM fuel is the heterogeneity of TRISO particles and the heterogeneity of fuel rods. The two types of heterogeneity are treated separately.

The self-shielding effect of TRISO particles is treated by equivalent homogenization method. In this method, the disadvantage factors are calculated firstly, and then the average cross sections over the TRISO particles and matrix calculated by volume weight are corrected using the disadvantage factors. Due to small changes in the cross section of fast-energy range, the self-shielding effect in TRISO particles is weak. So the self-shielding effect of fast-energy range in TRISO particles don't need to be treated. It is known that the resonance cross sections of the heavy isotopes in the fuel will cause the self-shielding effect of TRISO particles. Some studies have proven that the spatial self-shielding in TRISO particles will also occur in the thermal-energy range, due to the large absorption cross sections of the fuel in thermal-energy range (Hansen and Teuchert, 1971). As a result, the effective cross sections in this energy range cannot be directly generated by volume weight. The self-shielding effect of TRISO particles should be solved in resonance-energy and thermal-energy ranges.

After the equivalent homogenization, the FCM fuel can be treated as conventional PWR lattice. In this paper, the Dancoff factors are firstly calculated by neutron current method (Sugimura and Yamamoto, 2006) to treat the heterogeneity of fuel rods. A one-dimensional rod model is established for each fuel rod in the lattice by preserving the Dancoff factor. Finally, the hyperfine group calculation is carried out based on the one-dimensional rod model to obtain the effective cross sections of the fuel rod.

2.2. Treatment of self-shielding effect of TRISO particles

In equivalent homogenization method, the equivalent cross sections should preserving the reaction rates:

$$\tilde{\Sigma} V \bar{\phi} = \sum_i V_i \phi_i \Sigma_i \tag{1}$$

where $\tilde{\Sigma}$ is the equivalent cross section of the whole system; V is the volume of the whole system; V_i is the volume of the ith region; ϕ_i the flux of the ith region; Σ_i is the cross section of the ith region.

A transformation of Eq. (1) leads to the following form:

$$\tilde{\Sigma} = \sum_i f_i S_i \Sigma_i = \sum_i f_i \tilde{\Sigma}_i \tag{2}$$

and

$$S_i = \frac{\phi_i}{\bar{\phi}} \tag{3}$$

where f_i is the volume ratio of the ith region (i.e., V_i/V); S_i is the disadvantage factor of the ith region; $\tilde{\Sigma}_i$ is the equivalent cross section of the ith region and it can be described as

$$\tilde{\Sigma}_i = S_i \Sigma_i \tag{4}$$

It is obvious that the disadvantage factor reflects the flux depression in every sub-region caused by the spatial self-shielding effect in TRISO particles. When the disadvantage factor

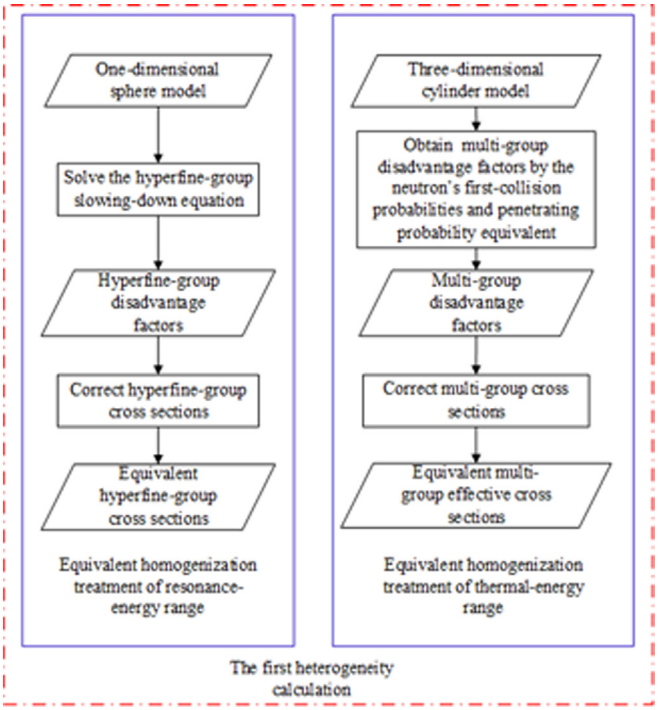

Fig. 1. Calculation flowchart of the first heterogeneity's treatment of FCM fuel.

is closer to unity, the spatial self-shielding effect is weaker and vice versa. Thus, it is very essential to compute the disadvantage factors.

The disadvantage factors in the resonance-energy range and thermal-energy range are calculated by different strategies. The detailed calculation flowchart is shown in Fig. 1. In resonance-energy range, the disadvantage factors are obtained by solving a one-dimensional model of the TRISO particle with hyperfine group method. It is difficult to obtain the disadvantage factors by solving hyperfine group slowing-down equation in the thermal-energy range, because iterative calculations are needed to consider the up-scattering of thermal neutron, which is time-consuming. Therefore, the multi-group disadvantage factors are used in thermal-energy range, which are calculated by the neutron's first-collision probabilities and penetrating probability equivalent proposed in She et al. (2017).

2.2.1. Calculation of disadvantage factor in resonance-energy range

In resonance-energy range, the disadvantage factors are obtained by solving a one-dimensional sphere model with hyperfine group method. To build the one-dimensional sphere model, an infinite stochastic media is assumed and the self-shielding between fuel rods is ignored. The infinite stochastic media consists of matrix material and only one type of TRISO particle.

As shown in Fig. 2, the one-dimensional sphere model is composed of the TRISO particle and the matrix outside of it. Employing the Wigner-Seitz equivalent cell approximation, the radius of the matrix in the one-dimensional model is

$$R_m = \frac{R_t}{\sqrt[3]{F}} \tag{5}$$

where R_m is the radius of the matrix in the one-dimensional model; R_t is the radius of the TRISO particle; F is the packing fraction (PF) of the TRISO particle.

The hyperfine group neutron slowing-down equation is given as

$$\Sigma_{ig}\phi_{ig}V_i = \sum_j V_j Q_{jg} P_{jig} \tag{6}$$

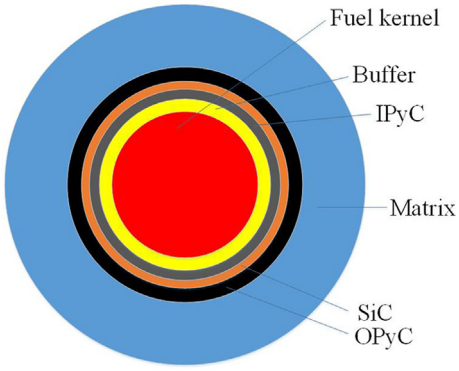

Fig. 2. One-dimensional sphere model for the calculation of disadvantage factors in resonance-energy range.

where Σ_{ig} is the macroscopic total cross section of the ith region; ϕ_{ig} is the volume averaged flux in the ith region; Q_{jg} is the volume averaged slowing-down source in the ith region; P_{jig} is the collision probability that a neutron born in the jth region has its first collision in the ith region; V_i and V_j are the volume of the ith and the jth region, respectively.

To speed up the calculation, the source term in Eq. (6) can be obtained by the following recursive equation (Leszczynski, 1987):

$$Q_{jg} = \exp(-\Delta u_f)Q_{j,g-1} + \sum_k \Sigma_{sjk,g-1}P_{1k}\phi_{j,g-1} - \exp(-\Delta u_f) \times \left(\sum_k \Sigma_{sjk,g-N_k-1}P_{N_k,k}\phi_{j,g-N_k-1}\right) \tag{7}$$

where j is the region index; Q_{jg} and $Q_{j,g-1}$ are the volume averaged slowing-down source of the gth group and (g-1)th group, respectively; Δu_f is the lethargy width; $\Sigma_{sjk,g-1}$ and $\Sigma_{sjk,g-N_k-1}$ are the macroscopic scattering cross section of the isotope "k" in the gth group and $(g-N_k-1)$th group, respectively; N_k is the maximum energy group that a neutron can traverse after a collision with the isotope "k"; P_{1k} and $P_{N_k,k}$ are the probability that a neutron traverses one energy group and N_k energy groups after a collision with the isotope "k", respectively; $\phi_{j,g-1}$ and $\phi_{j,g-N_k-1}$ are the volume averaged flux of the (g-1)th group and $(g-N_k-1)$th group, respectively. The P_{nk}(n = 1, N_k) are obtained by

$$P_{nk} = A\exp\{-(n-1)\Delta u_f\}\{1-\exp(-\Delta u_f)\}^2 \tag{8}$$

where $A = 1/\{(1-\alpha_k)\Delta u_f\}$:$\alpha_k = (A_k-1)^2/(A_k+1)^2$; A_k is the mass of the isotope "k", in units of the neutron mass.

To calculate the sources Q_{jg} in Eq. (6), the initial sources can be set as a constant arbitrarily. The collision probabilities in Eq. (6) can be obtained by Carlvik integration (Carlvik, 1964). After the flux of every sub-region is obtained, the hyperfine group disadvantage factors are calculated by Eq. (3).

In the researches of HTR fuel pebbles, there exist such designs where a pebble element simultaneously contains fuel particles and burnable poison particles (She et al., 2017). To make the proposed method can also be applied in this type of design for FCM fuel, the treatment for multiple particle types are also considered. In order to obtain hyperfine group disadvantage factors of the stochastic media with multiple particle types, it is assumed that the shadowing effect between particles can be neglected. Each particle type is calculated independently with one-dimensional model. The numerical results show that this approximation will not introduce obvious error to the results of multi-group effective cross sections and infinite multiplication factors. The reason is that the neutron average free path is less than the distance between the

TRISO particles so that the shadowing effect among particles is very weak.

A problem needed to be discussed is the computation efficiency, because the hyperfine group method is adopted in the calculation. In the calculations, the more computation regions, the lower the computational efficiency. Therefore, the matrix and the silicon carbide shell can be homogenized by volume weight to accelerate the calculation of hyperfine group method, and it will not introduce obvious error. Thus, with the matrix and silicon carbide shell homogenized, one-dimensional particle model contains only two regions, fuel region and non-fuel region, respectively. The calculation time of this two region model with 39,200 hyperfine group is about 1 s. There are some other techniques which can also be used to improve the calculation efficiency further. For instance, the collision probability can be obtained by interpolation method; the scattering source of moderator can be calculated with $1/E$ spectrum (Sugimura and Yamamoto, 2007). Therefore, the proposed method can be used in the practice double heterogeneity treatment.

After the disadvantage factors are obtained by hyperfine group method, the matrix and TRISO particles will be homogenized by correcting the hyperfine group cross sections according to Eq. (4).

2.2.2. Calculation of disadvantage factor in thermal-energy range

In order to calculate the disadvantage factors in thermal-energy range, a three-dimensional cylinder model is built, as shown in Fig. 3. Considering that a neutron beam traverses a distance L in the stochastic media, it either takes its first collision in a certain sub-region or has no collision along the trajectory. The distance L can be derived as

$$\frac{4}{3}\pi R_t^3 = \pi R_t^2 L \cdot F \tag{9}$$

and

$$L = \frac{4R_t}{3F} \tag{10}$$

Suppose that p_i is the neutron's first-collision probabilities in the ith sub-region and p is the probability that a neutron penetrates a distance L without collision. Obviously, the sum of these probabilities should be exactly unity:

$$p + \sum_i p_i = 1 \tag{11}$$

Assuming that the stochastic media is transformed into an equivalent homogeneous media, the penetrating probability p can be written as $p = e^{-\tilde{\Sigma}L}$, where $\tilde{\Sigma}$ is the equivalent cross section. When p is known, $\tilde{\Sigma}$ can be obtained by

$$\tilde{\Sigma} = -\frac{\ln(p)}{L} \tag{12}$$

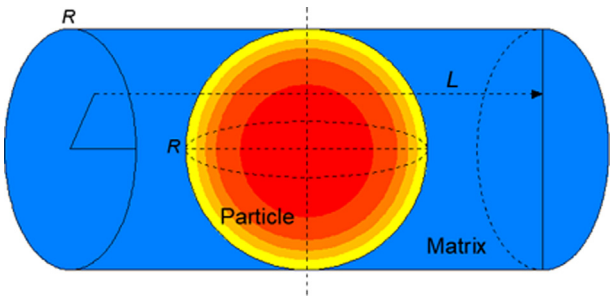

Fig. 3. Three-dimensional cylinder model for the calculation of disadvantage factors in thermal-energy range.

According to Eq. (11), the neutron's first-collision probabilities in all sub-regions is 1-p, and the proportion contributed by the ith sub-region is $\frac{f_i\tilde{\Sigma}_i}{\tilde{\Sigma}}$, as indicated in Eq. (2). As a result, the first-collision probability in the ith sub-region is

$$p_j = (1-p)\frac{f_i\tilde{\Sigma}_i}{\tilde{\Sigma}} \tag{13}$$

By simple transforming, the Eq. (13) becomes

$$\tilde{\Sigma}_i = \tilde{\Sigma}\,\frac{p_i}{1-p}\frac{1}{f_i} \tag{14}$$

It can be seen that if the neutron's first-collision probabilities in every sub-regions and penetrating probability are known, $\tilde{\Sigma}_i$ can be calculated easily. The method to calculate these probabilities can be found in She et al. (2017). Finally, the disadvantage factor in the ith sub-region can be obtained by Eq. (4) after $\tilde{\Sigma}_i$ are obtained. As soon as the disadvantage factors are known, the equivalent cross section of all reaction types can be immediately solved from Eq. (4).

Equivalent homogenization treatment of the stochastic media with multiple particle types in thermal-energy range is carried out just as the treatment in resonance-energy range mentioned above.

2.3. Treatment of self-shielding effect of fuel rods in the lattice system

After the homogenization of the TRISO particles and matrix in the fuel rod, the lattice can be treated as the typical PWR lattice. The heterogeneity of fuel rods is described by Dancoff correction factor. In this paper, the Dancoff correction factor is calculated using neutron current method. The Dancoff correction factor is calculated as follows:

$$C = \frac{\phi_0 - \phi}{\phi_0} \tag{15}$$

where ϕ_0 is the flux of fuel region in an isolated rod; ϕ is the flux of fuel region in a lattice system. The flux for each fuel rod in a lattice system is obtained by solving a one-group fixed-source equation established based on the real geometry of the lattice. And the flux in the fuel region in an isolated rod is also obtained by solving a one-group fixed source equation established based on the isolated rod system. The fixed source equations can be efficiently solved by method of characteristics (MOC) based code.

For a one-dimensional isolated rod, the Dancoff correction factor can be calculated by

$$C = \frac{P_e - P_{f\to m}(R)}{\Sigma_{t,f}\,\tilde{l}\,P_{f\to m}(R) + P_e - P_{f\to m}(R)} \tag{16}$$

where R is the radius of moderator; P_e is the neutron escape probability from the fuel; $P_{f\to m}(R)$ is the collision probability that a neutron born in the fuel has its first collision in the moderator; $\tilde{l}$ is the average chord length of the fuel rod.

In Eq. (16), when the geometry and material information are given, P_e, $\tilde{l}$ and $\Sigma_{t,f}$ can be easily obtained. It can also be seen that the Dacoff correction factor of the isolated fuel rod is dependent on the moderator radius, because $P_{f\to m}(R)$ is the function of the moderator radius. Therefore, after the Dancoff correction factors for each fuel rod in the lattice are obtained using neutron current method, an equivalent one-dimensional rod can be established for each fuel rod by changing the moderator radius to preserve the Dancoff correction factor in the two system. In the realistic calculations, the moderator radius is determined by dichotomy.

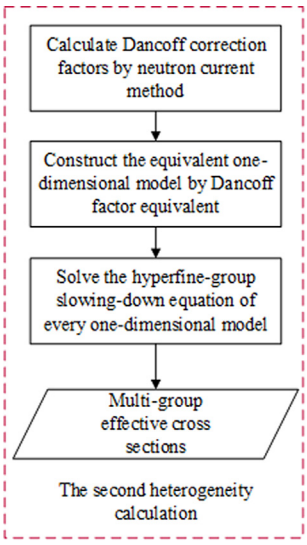

Fig. 4. Calculation flowchart of the second heterogeneity's treatment of FCM fuel.

Finally, the hyperfine group method introduced in Section 2.2.1 is adopted to calculate multi-group effective cross sections for each equivalent one-dimensional fuel rod. Since the equivalent fuel rods are independent on each other, the hyperfine group calculation can be carried out in parallel. The detailed calculation flowchart of the self-shielding treatment for fuel rods is shown in Fig. 4.

3. Numerical results

The problems of fuel-pin cells, burnable-poison-pin cells and 2D lattice are calculated to test the precision of the proposed multi-group effective cross section calculation method. Test problems are from Awan et al. (2017). In the tests, the reference results are provided by the continuous energy Monte Carlo (MC) code which uses regular lattice model. The relative statistical error of flux and reaction rate is $\sim 10^{-3}$. The multi-group structure is WIMSD 69 group format. The range of resonance-energy group is from group 13–45.

3.1. Assessment of DH modelling approaches in MC code

In the MC code, there are mainly three types of DH modelling approaches, explicit stochastic model, implicit model and regular lattice model, respectively. To test the differences between different models, a fuel-pin cell case is designed. The configuration of fuel-pin cell is shown in Fig. 5. The temperature of all regions in the fuel-pin cell is 600 K. The geometry parameters and material composition of the fuel-pin cell are listed in Table 1. The geometry parameters and material composition of TRISO particle are listed in Table 2. The atomic number density for each material is listed in Table 3. The PF of TRISO particles is 35%. The results from explicit stochastic model are considered as reference solution.

The relative differences of the effective absorption cross sections for U-238 for the fuel-pin cell case are shown in Fig. 6. It is observed that

1. There are certain differences between all different models.
2. The relative differences between implicit model and other model are reached to 3.2%.

The explicit stochastic model is generally considered as a relatively accurate model. However, the algorithm for the random sequential packing does not produce random realization for PFs greater than approximately 0.40. The relative differences between implicit model and explicit stochastic model are so large that it is not suitable for a reference solution model. As a result, the regular lattice model is chosen in this paper.

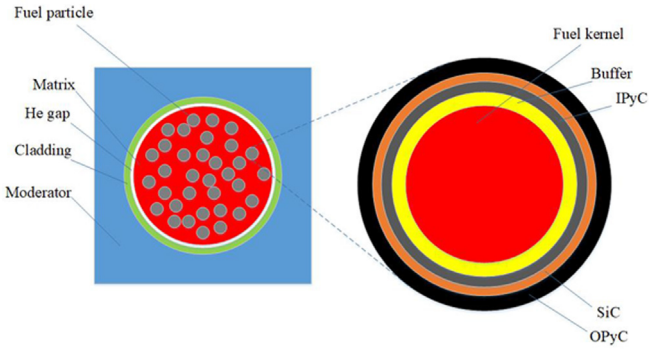

Fig. 5. FCM pin cell.

Table 1
Dimensions and materials of FCM cell.

Region	Radius or pitch (cm)	Material
Fuel lump	0. 6252	TRISO particles + SiC
Gap	0.6337	He
Cladding	0.6907	FeCrAl
Moderator	1.65	H_2O

Table 2
Dimensions and materials of fuel particle.

Material	Radius (cm)
Kernel	0.0250
Buffer	0.0340
IPyC	0.0380
SiC	0.0415
OPyC	0.0455

Table 3
Atomic number density for each material.

Material	Nuclide	Atom density (/barn-cm)
Kernel (14.3%)	U-235	4.51628E-3
	U-238	2.67239E-2
	C-12	3.12402E-2
Kernel (10%)	U-235	3.15987E-3
	U-238	2.80803E-2
	C-12	3.12402E-2
Kernel (5%)	U-235	1.58094E-3
	U-238	2.96593E-2
	C-12	3.12402E-2
Buffer	C-12	5.26455E-2
IPyC/OPyC	C-12	9.52634E-2
SiC	C-12	4.77618E-2
	Si-28	4.40507e-2
	Si-29	2.23048e-3
	Si-30	1.48062e-3
He	He-4	2.6900E-5
FeCrAl	Fe-54	7.99520E-4
	Fe-56	1.22593E-2
	Fe-57	2.66507E-4
	Cr-52	3.55342E-3
	Al-27	8.88356E-4
H_2O	H-1	4.86165E-2
	O-16	2.43083E-2
B_4C	B-10	1.52689E-02
	B-11	6.14591E-02
	C-12	1.91820E-02

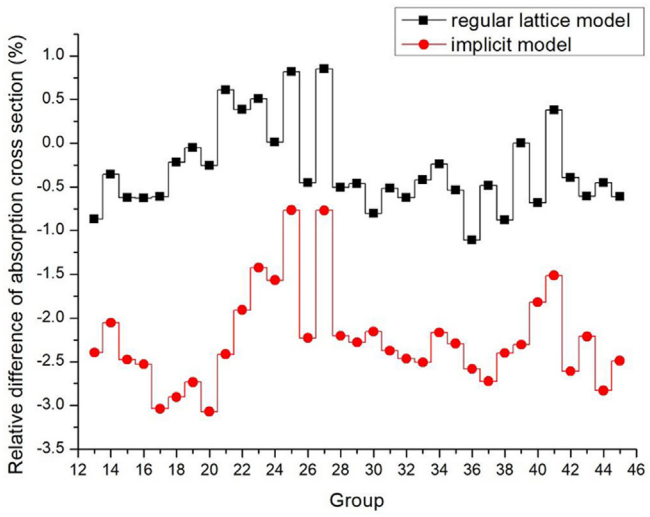

Fig. 6. Relative differences of effective absorption cross sections of U-238 between different models.

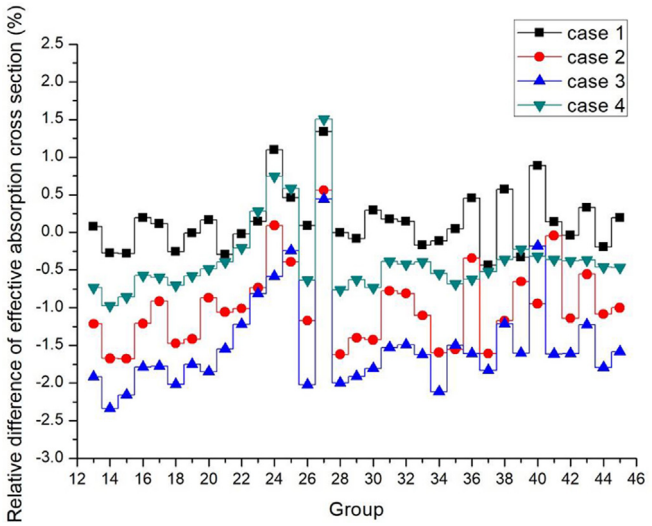

Fig. 7. Relative differences of effective absorption cross sections of U-238 in FCM pin cell.

3.2. Verification of fuel-pin cell problem

Due to the peculiarity of double heterogeneous fuel, many parameters, such as particle PF, fuel kernel radius, fuel enrichment, fuel temperature and so on, significantly affect the result (Kim et al., 2017; Choi et al., 2015). Therefore, a series of test cases are designed to test the sensitivity of the proposed method. Table 4 summarizes all the cases for the sensitivity test of the proposed method.

The configuration of fuel-pin cell is shown in Fig. 5. The temperature of all regions in the fuel-pin cell is the same. The material of matrix is SiC. The geometry parameters and material composition of the fuel-pin cell are listed in Table 1. The geometry parameters and material composition of TRISO particle are listed in Table 2. The atomic number density for each material is listed in Table 3.

Cases 1–4 are used for the sensitivity test of PFs for the proposed method. The relative differences of the effective absorption cross sections for U-238 and U-235 in the fuel-pin cell cases from 1 to 4 are shown in Figs. 7 and 8, respectively. In case 1, 2 and 4, almost all groups have small differences less than 1.00% and the maximum difference is 1.7% for U-238. For U-235, almost all groups have small differences less than 1.00% and the maximum difference is 1.6%. In case 3, the maximum difference is 2.3% for U-238 and 2.1% for U-235 and the differences are negative in general. As mentioned in the previous section, the different DH models in MC code also have differences in some extent. So it is considered that the differences in case 3 are caused by the DH model in the MC code. In the thermal-energy range, there exists strong spatial self-shielding effect in the TRISO particles, which will introduce large error if the effective cross sections are calculated by volume weight, as shown in Fig. 9. When the effective cross sections are corrected by the disadvantage factors, the differences are significantly reduced. For group 46–68, the largest difference is 2.2%. For the last group, the maximum difference is 4.8%, which is mainly caused by the error of multi-group cross section library processed by NJOY, because there exists deviation of the theoretical model in obtaining multi-group cross section library in NJOY.

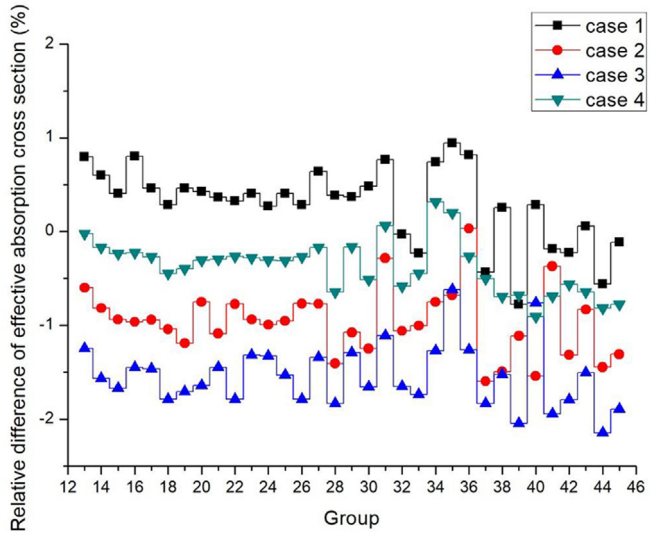

Fig. 8. Relative differences of effective absorption cross sections of U-235 in FCM pin cell.

Table 4
Cases for sensitivity test.

Case number	Fuel temperature (K)	Fuel enrichment (%)	Kernel radius (μm)	Grain PF (%)
1	600	14.3	250	35
2	600	14.3	250	40
3	600	14.3	250	45
4	600	14.3	250	48
5	300	14.3	250	40
6	600	10.0	250	40
7	600	5.0	250	40
8	600	14.3	300	40

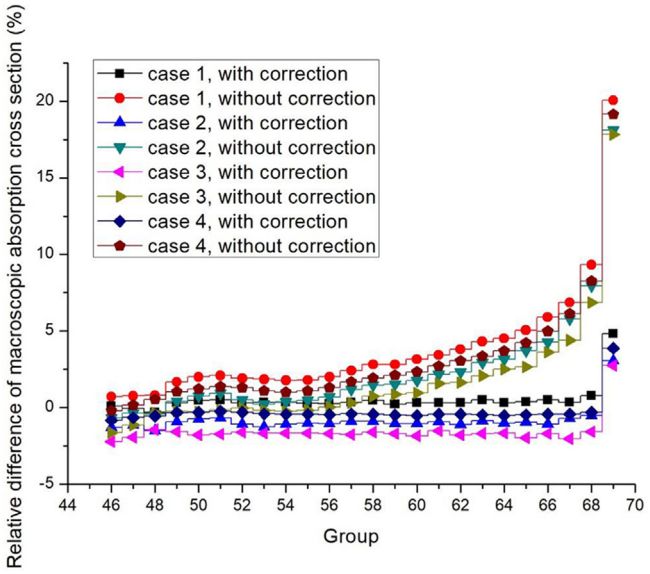

Fig. 9. Relative differences of macroscopic cross sections of fuel lump in FCM pin cell.

To test the sensitivity of temperatures, case 5 is designed. The relative differences of the effective absorption cross sections for U-238 and U-235 in the fuel-pin cell cases 2 and 5 are shown in Figs. 10 and 11, respectively. For U-238, almost all groups have small differences less than 1.00% and the maximum difference is 1.7%. For U-235, almost all groups have small differences less than 1.00% and the maximum difference is 1.6%. The differences of effective cross sections in thermal-energy range are shown in Fig. 12. The effective cross sections corrected by disadvantage factors agree well with the reference results.

Case 6 and 7 are used for the sensitivity test of fuel enrichments. The relative differences of the effective absorption cross sections for U-238 and U-235 in the fuel-pin cell cases 2, 6 and 7 are shown in Figs. 13 and 14, respectively. For U-238, almost all groups have small differences less than 1.00% and the maximum difference is 1.7%. For U-235, almost all groups have small differences less than 1.00% and the maximum difference is 1.6%. The differences of effective cross sections in thermal-energy range are shown in Fig. 15.

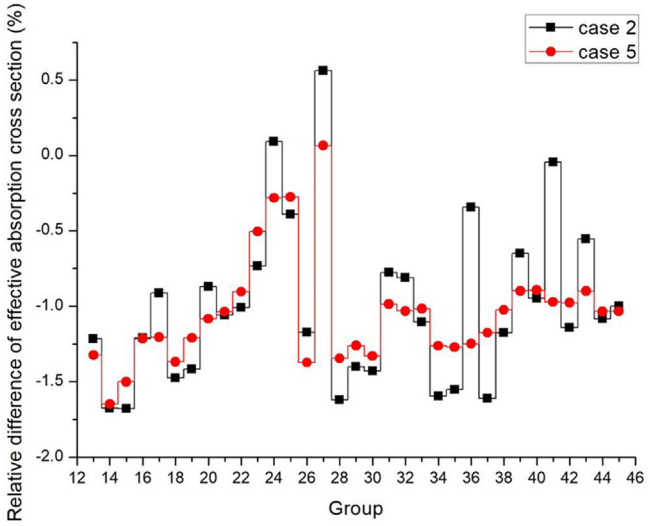

Fig. 10. Relative differences of effective absorption cross sections of U-238 in FCM pin cell.

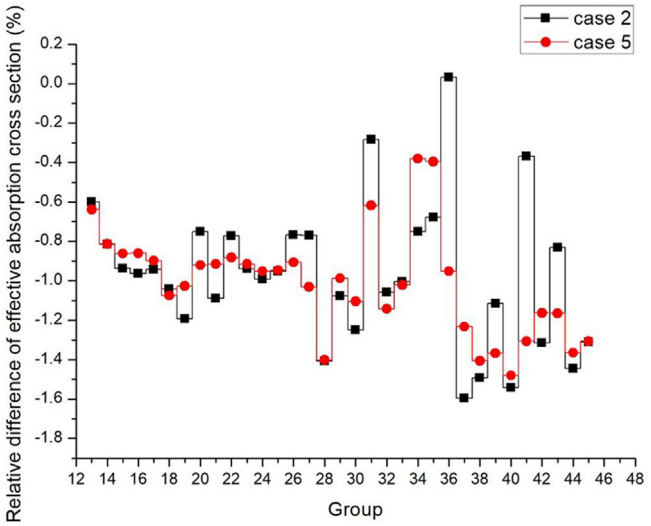

Fig. 11. Relative differences of effective absorption cross sections of U-235 in FCM pin cell.

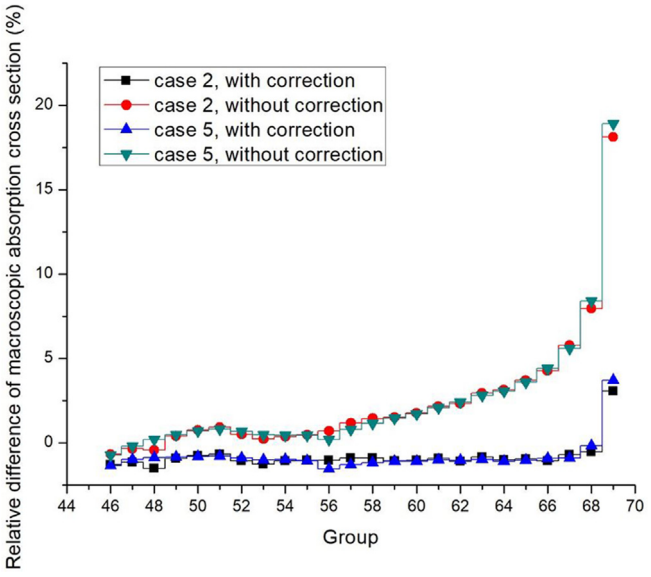

Fig. 12. Relative differences of macroscopic cross sections of fuel lump in FCM pin cell.

The effective cross sections corrected by disadvantage factors agree well with the reference results.

Case 8 is used for the sensitivity test of fuel kernel radius. The relative differences of the effective absorption cross sections for U-238 and U-235 in the fuel-pin cell cases 2 and 8 are shown in Figs. 16 and 17, respectively. For U-238, almost all groups have small differences less than 1.00% and the maximum difference is 1.7%. For U-235, almost all groups have small differences less than 1.00% and the maximum difference is 1.6%. The differences of effective cross sections in thermal-energy range are shown in Fig. 18. The effective cross sections corrected by disadvantage factors agree well with the reference results.

Table 5 gives the results of the infinite multiplication factors of the fuel-pin cell cases from 1 to 8. In addition to case 7, the relative differences of these fuel-pin cell cases are less than 121 pcm. In can be seen that the differences can be significantly reduced with correction by the disadvantage factors. So it is concluded that the effective cross sections in thermal-energy range should be

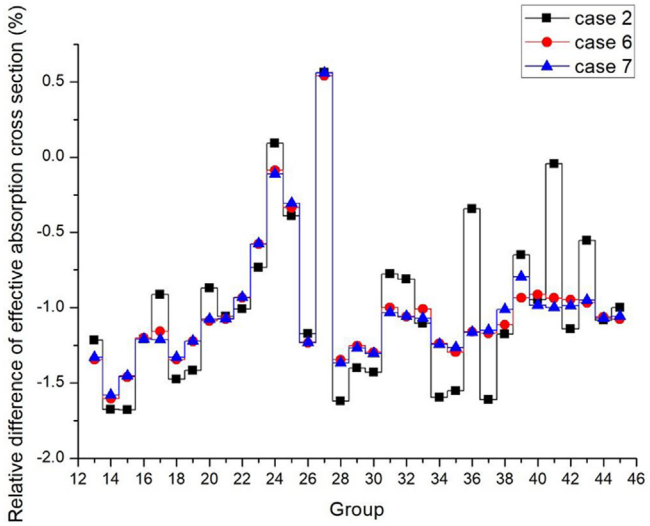

Fig. 13. Relative differences of effective absorption cross sections of U-238 in FCM pin cell.

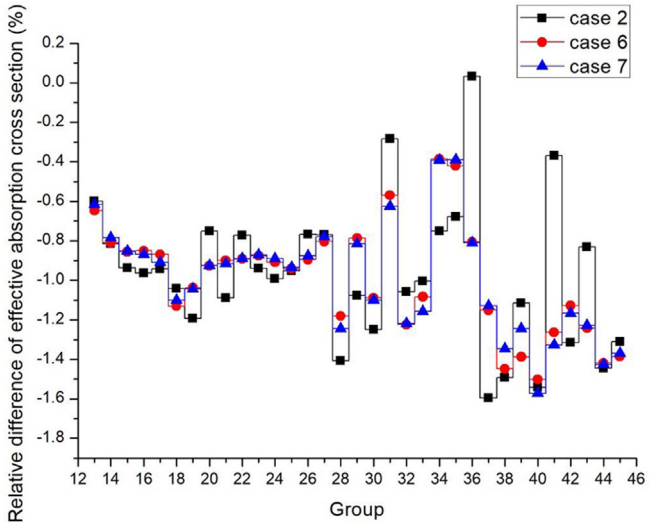

Fig. 14. Relative differences of effective absorption cross sections of U-235 in FCM pin cell.

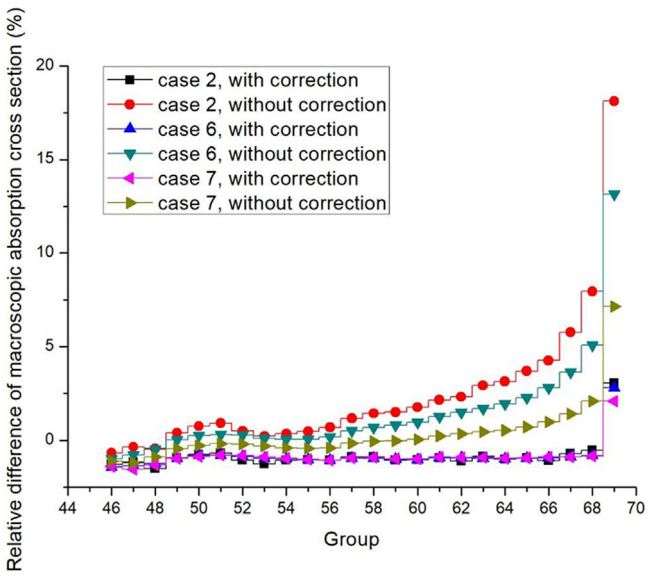

Fig. 15. Relative differences of macroscopic cross sections of fuel lump in FCM pin cell.

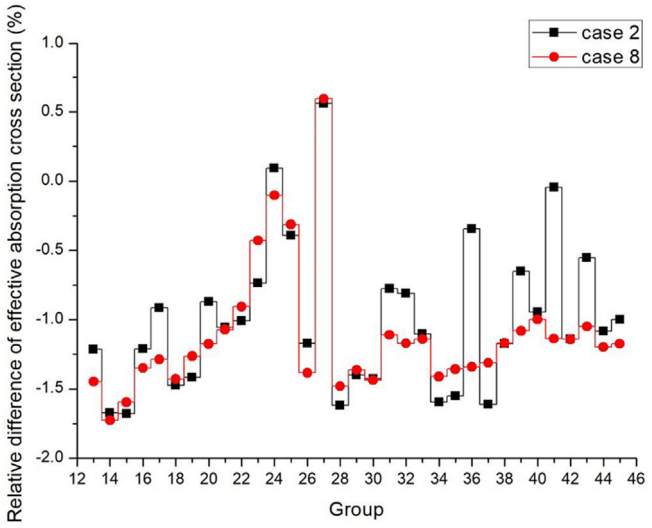

Fig. 16. Relative differences of effective absorption cross sections of U-238 in FCM pin cell.

corrected by disadvantage factors to describe the self-shielding effect of TRISO particles. The relative difference of case 7 is 228 pcm with correction and 104 pcm without correction. Due to weak self-shielding effect in low fuel enrichment, it can also achieve good result without correction.

The time of resonance calculation for a fuel-pin cell is about 10 s. The time is completely acceptable and far less than the calculation time of the MC code.

3.3. Verification of burnable-poison-pin cell problem

Two types of burnable-poison-pin cells are calculated in this section. The first type of burnable-poison-pin cell only contains one type of burnable poison particle. The PF of burnable poison particle is 40%. The configuration of the burnable poison particle is shown in Fig. 19. The geometry parameters and materials of the burnable poison particle are listed in Table 6. The second type of burnable-poison-pin cell contains burnable poison particles and fuel particles. The PF of burnable poison particle and fuel particle both are 17%. The configuration of the burnable poison particle is shown in Fig. 20. The configuration of the fuel particle is the same as the fuel particle in pin cell mentioned above. The geometry parameters and materials of the burnable poison particle in the second type of burnable-poison-pin cell are listed in Table 7. The temperature of all the materials in the two problems is 600 K. The atomic number density for each material is listed in Table 3. Due to the limitation of regular lattice model in MC code, the results from explicit stochastic model is chosen as the reference solution for the second type of burnable-poison-pin cell.

The relative differences of the effective absorption cross sections for U-238 and U-235 for the first case are shown in Figs. 21 and 22, respectively. For U-238, almost all groups have small differences less than 1.00% and the maximum difference is 2.0%. For U-235, almost all groups have small differences less than 1.00% and the maximum difference is 1.7%. The differences of effective cross sections in thermal-energy range are shown in Fig. 23. It can be seen that for the burnable TRISO particles, lager error will

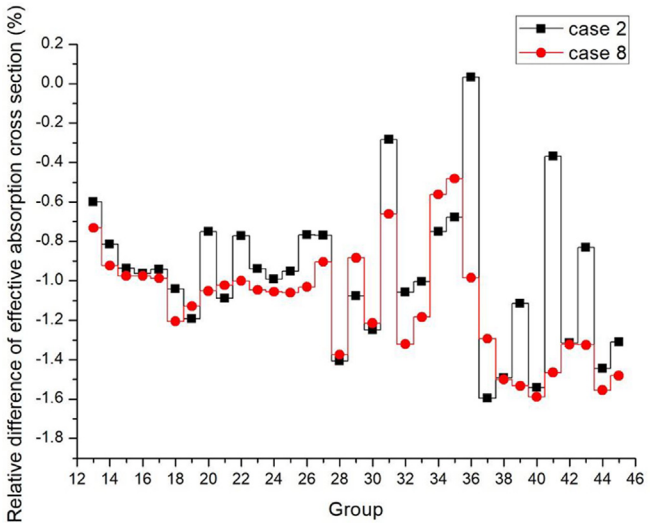

Fig. 17. Relative differences of effective absorption cross sections of U-235 in FCM pin cell.

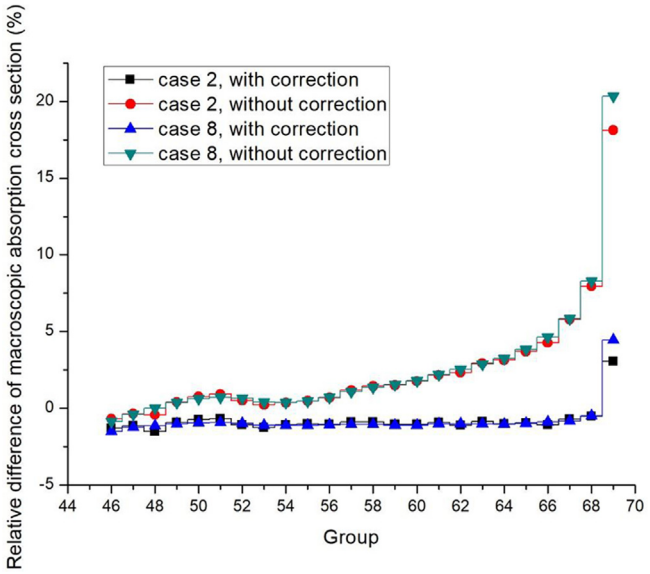

Fig. 18. Relative differences of macroscopic cross sections of fuel lump in FCM pin cell.

be introduced than the fuel pin if the spatial self-shielding effect in the particles are ignored, while the effective cross sections corrected by disadvantage factors agree well with the reference results. Table 8 gives the results of the infinite multiplication

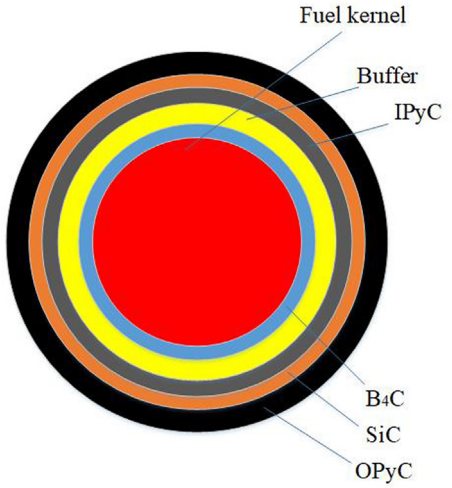

Fig. 19. First type of burnable poison particle.

Table 6
Dimensions and materials of the first type of burnable poison particle.

Material	Radius (cm)
Kernel	0.0242
B_4C	0.0250
Buffer	0.0340
IPyC	0.0380
SiC	0.0415
OPyC	0.0455

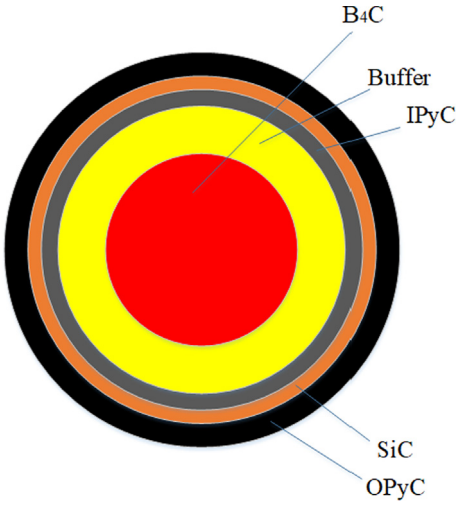

Fig. 20. Second type of burnable poison particle.

Table 5
Results and relative differences of k-inf of FCM fuel cells.

Case number	Reference value[a]	With disadvantage factors in thermal-energy range		Without disadvantage factors in thermal-energy range	
		Calculation value	Relative differences[b] (pcm)	Calculation value	Relative differences[b] (pcm)
1	1.53851	1.53912	40	1.54554	457
2	1.55902	1.55759	−69	1.56320	268
3	1.57252	1.57065	−119	1.57560	196
4	1.57768	1.57662	−67	1.58123	225
5	1.57725	1.57605	−76	1.58304	367
6	1.46785	1.46607	−121	1.47132	236
7	1.22679	1.22399	−228	1.22807	104
8	1.58799	1.58676	−77	1.59070	171

[a] The relative statistical error is less than 1 pcm.
[b] Relative differences = (Calculation value − Reference value)/Reference value.

Table 7
Dimensions and materials of the second type of burnable poison particle.

Material	Radius (cm)
B_4C	0.0090
Buffer	0.0340
IPyC	0.0380
SiC	0.0415
OPyC	0.0455

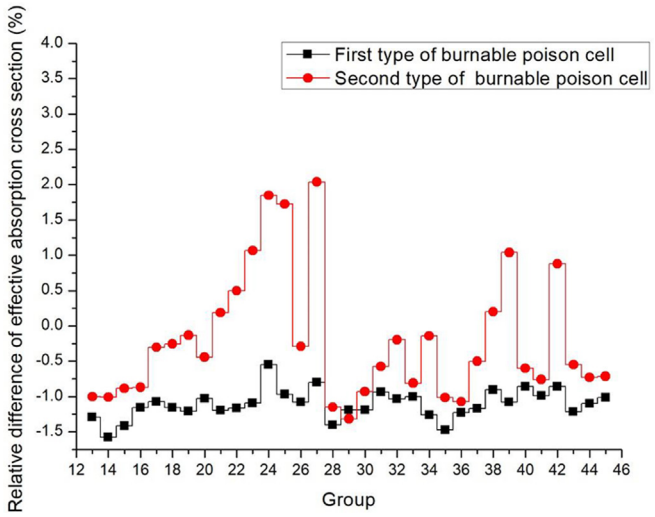

Fig. 21. Relative differences of effective absorption cross sections of U-238 in FCM burnable poison cells.

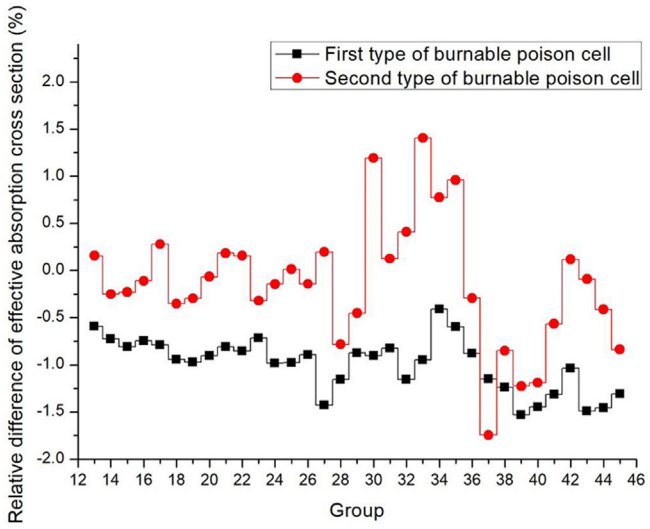

Fig. 22. Relative differences of effective absorption cross sections of U-235 in FCM burnable poison cells.

factors of the two types of burnable-poison-pin cells. It can be seen that the relative differences of the two types of burnable-poison-pin cells are very small.

3.4. Verification of 2D lattice problem

The configuration of 2D lattice is shown in Fig. 24. The fuel pin cell in 2D lattice is described in Section 3.2. The geometry parameters and materials of guide tube cell are listed in Table 9. The atomic number density for each material is listed in Table 3. The PF of TRISO particles is 45%. The temperature of all regions in the 2D lattice is 600 K.

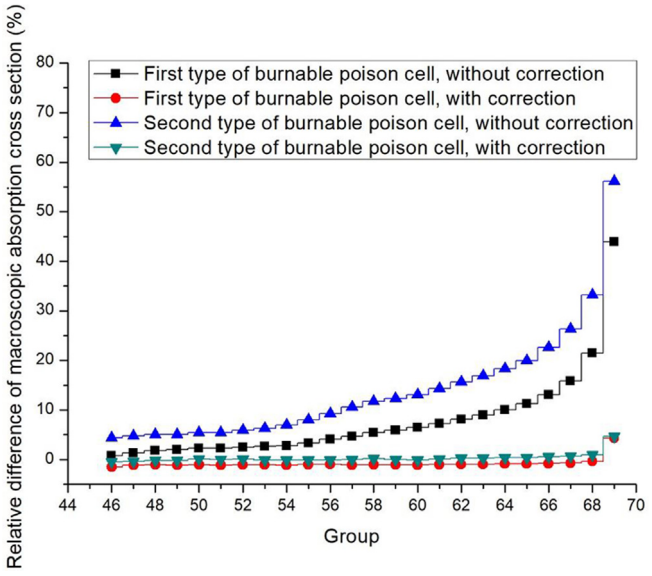

Fig. 23. Relative differences of macroscopic cross sections of fuel lump in FCM burnable poison cells.

Table 8
Results and relative differences of k-inf of FCM burnable poison cells.

Type	Calculation value	Reference value	Relative differences (pcm)
1	0.59105	0.59055	−85
2	0.89002	0.89012	−11

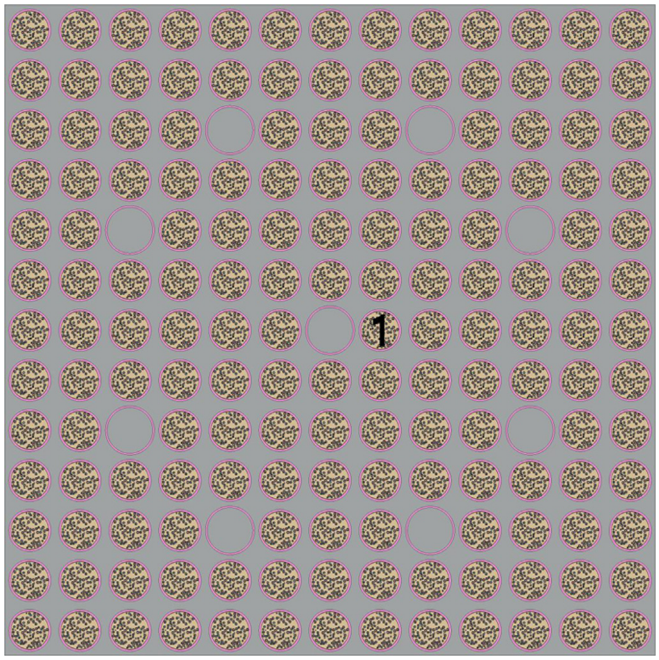

Fig. 24. 2D FCM lattice.

Table 9
Dimensions and materials of guide tube.

Region	Radius or pitch (cm)	Material
Moderator	0. 7555	H_2O
Guide tube	0.8055	FeCrAl
Moderator	1.65	H_2O

A fuel pin signed in Fig. 24 is analyzed. The relative differences of the effective absorption cross sections for U-238 and U-235 for the pin 1 are shown in Figs. 25 and 26, respectively. For U-238, almost all groups have small differences less than 1.00% and the maximum difference is 1.6%. For U-235, almost all groups have small differences less than 1.00% and the maximum difference is 1.7%. As shown in Fig. 27, the correction method also shows good accuracy in thermal-energy range. The result of the infinite multiplication factor of the 2D lattice is given in Table 10. It can be seen that the relative difference of the 2D lattice is very small.

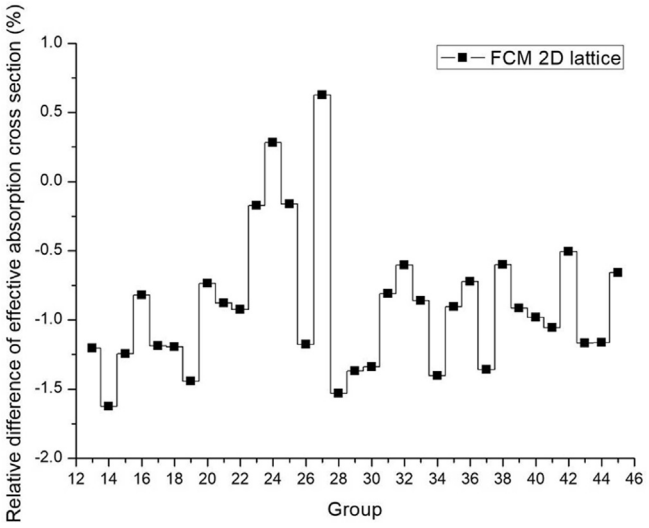

Fig. 25. Relative differences of effective absorption cross sections of U-238 in FCM lattice.

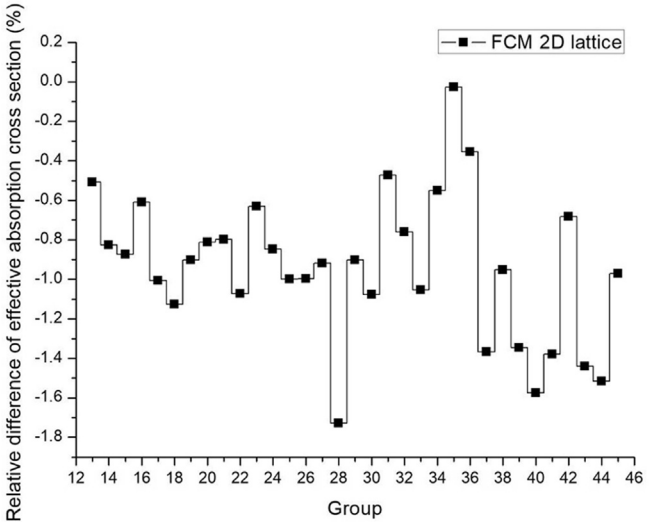

Fig. 26. Relative differences of effective absorption cross sections of U-235 in FCM lattice.

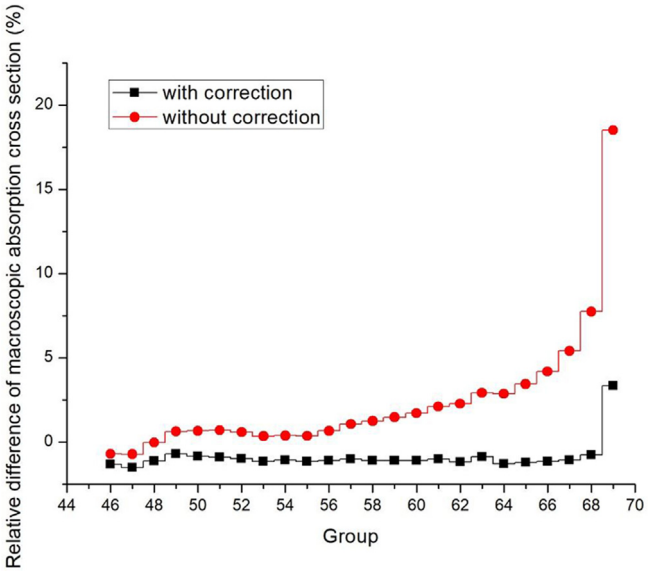

Fig. 27. Relative differences of macroscopic cross sections of fuel lump in FCM lattice.

Table 10
Result and relative difference of k-inf of FCM lattice.

Calculation value	Reference value	Relative difference (pcm)
1.56805	1.56995	−121

4. Conclusions

A new method to calculate multi-group effective cross section of FCM fuels is proposed in this paper. The double heterogeneity is separated into two parts and they are solved respectively. The heterogeneity of TRISO particles is solved by equivalent homogenization method. In this step, the disadvantage factors in resonance-energy and thermal-energy ranges are calculated using different methods. After the equivalent homogenization, the heterogeneity of the fuel pins can be treated as current PWR lattice. In the present work, a method coupling neutron current method and hyperfine group method is adopted to calculate the effective cross sections of the homogenized fuel pins.

Several different type of DH questions are tested in this work and numerical results show that the effective cross sections and eigenvalues obtained from the new method agree well with reference values from Monte Carlo code. The proposed method is capable of treating the double heterogeneity of FCM fuels, and providing exact multi-group effective cross sections.

For the thermal-energy range, the spatial self-shielding effect in the TRISO particles, especially in burnable poison particles, must be dealt with to obtain exact multi-group effective cross sections. Numerical results show that exact multi-group effective cross sections in thermal-energy range can be obtained by correcting multi-group effective cross sections with disadvantage factors.

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